"Instead of real time t, consider imaginary time τ" (Arthur S. Eddington)

A Spacetime Surprise: Time Isn't Just Another Dimension
By Ethan Siegel, August 12, 2020

Now, let's not simply consider space, but time as well. You might think, "well, if time is just a dimension, too, then the distance between any two points in spacetime will work the same way." For example, if we represent the time dimension as t, you might think the distance would be the straight line connecting two points through the three spatial dimensions as well as the time dimension. In mathematical terms, you might think that the equation for the separation between any two points would look something like $d = \sqrt{(x^2 + y^2 + z^2 + t^2)}$.

The speed of light in a vacuum — 299,792,458 meters per second — tells us precisely how to relate our motion through space with our motion through time: by that fundamental constant itself. When we use terms like "one light-year" or "one light-second," we're talking about distances in terms of time: the amount of distance that light travels in one year (or one second), for example. If we want to convert "time" into a distance, we need to multiply it by the speed of light in a vacuum.

But the second way requires an enormous leap to understand: something that eluded the greatest minds of the late 19th and early 20th centuries. The key idea is that we're all moving through the Universe, through both space and time, simultaneously. If we're simply sitting here, stationary, and not moving through space at all, then we move through time at a very specific rate at which we're all familiar: *one second per second*.

But there's an even deeper insight here, which initially eluded even Einstein himself. If you treat time as a dimension, multiply it by the speed of light, and — here's the big leap — treat it as though it were **imaginary**, rather than real, then we can define a "spacetime interval" the same way we defined distance earlier. Only, since the imaginary number **i** is just $\sqrt{(-1)}$, this means that the spacetime interval is actually $d = \sqrt{(x^2 + y^2 + z^2 - c^2t^2)}$. [Note the **minus sign** attached to the time coordinate!]

In other words, the transformation from "motion through or separation in space" to "motion through or separation in time" is also a rotation, but it's a rotation not in the cartesian coordinates of space (where x, y, and z are all real numbers), but through the hyperbolic coordinates of spacetime, where if the space coordinates are real, then **the time coordinate must be imaginary**.